2025

(May–June)

FYUP : 4th Semester Examination

ECONOMICS

(Mathematical Methods for Economics—II)

(ECO–251)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking at least **one** from each Unit

UNIT—I

1. (a) What is an idempotent matrix? Examine if the matrix A is idempotent

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

1+5=6

(b) If $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & 3 \\ 0 & 4 & 1 \end{bmatrix}$, then find $A^2 - 6A + 8I$. 6

(c) Define the meaning of a vector using a suitable example. Show that the vectors $V_1 = (1, 5, -3)$ and $V_2 = (2, -1, -1)$ are orthogonal. 2+1=3

2. (a) Explain any three properties of determinants providing examples. 6

(b) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ 4 & 1 \end{bmatrix}$, then show that $(ABC)' = C'B'A'$. 6

(c) Distinguish between a symmetric matrix and a skew-symmetric matrix. 3

3. (a) Solve the following equations by matrix inversion method : 9

$$x + y - z = 4$$
$$3x - 4z = 5$$
$$4x - 5y = 2$$

(b) (i) Find the rank of the matrix

$$\begin{bmatrix} 3 & 2 & 1 \\ 3 & 4 & 3 \\ -1 & 2 & 2 \end{bmatrix}$$

(ii) For what value of x, the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 7 & 0 & x \end{bmatrix}$ will be 2? 3+3=6

UNIT—II

4. (a) Define a homogeneous function. 2

(b) Determine the degree of homogeneity of the following functions (any *three*) : 3×3=9

(i) $f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3$

(ii) $f(x, y) = \dfrac{x^2 + y^2}{2x^2 y}$

(iii) $f(x, y) = (ax^{-3} + by^{-3})^{-\frac{1}{3}}$

(iv) $f(x, y) = \sqrt{ax^2 + bxy + cy^2}$

(v) $f(x, y, w) = x^4 - 5yw^3$

(c) Use the implicit function theorem to show that

$$x^2y^3 + 3xy^2 + y = 22$$

implies an explicitly defined function $y = f(x)$ at point $(x = 1, y = 2)$. Also find the value of derivative $\dfrac{dy}{dx}$ at this point. 4

5. (a) Under unconstrained optimization, what are the necessary and sufficient conditions of finding maxima and minima for a function $u = f(x, y)$? 3

(b) Determine maxima/minima for the function $z = x^2 + 3y^2 - xy - 6x - 12y + 10$. 5

(c) The total revenue and total cost functions of a firm producing two goods are

$$R = 10x + 5y$$
$$C = x^2 + xy + 2y^2$$

Find the profit maximizing levels of x and y. 7

6. (a) Using Lagrange multiplier, find the extreme value of the function $z = xy$ subject to the constraint $x + y = 50$. 6

(b) Solve for consumer equilibrium if the utility function is $U = x^{1.5}y$, price of commodity $x =$ ₹1, price of commodity $y =$ ₹4 and consumer's money income = ₹100. 9

UNIT—III

7. (a) Write down the order and degree of the following difference equations : 1+1=2

(i) $Y_t = 2Y_{t-1} - 3Y_{t-2} + 4$

(ii) $(Y_{x+2})^2 - Y_{x+1} + Y_x = 0$

(b) Solve the following difference equations using general method (any *three*) : 3×3=9

(i) $Y_{x+1} - Y_x = 0,\ Y_0 = 2$

(ii) $Y_{t+1} - \frac{1}{3}Y_t = 6,\ Y_0 = 1$

(iii) $10 - 2P_t = 3P_{t-1} - 5,\ P_0 = 12$

(iv) $Y_x - Y_{x-1} = 4,\ Y_0 = 4$

(c) If a market model has the numerical form

$$Q_{dt} = 10 - 2P_t$$
$$Q_{st} = -5 + 3P_{t-1}$$

find the inter-temporal equilibrium price and determine whether the equilibrium is stable or not. 4

8. (a) Solve the following differential equations (any *three*) : 3×3=9

(i) $\frac{dy}{dx} = e^{x-y} + xe^{-y}$

(ii) $\frac{dy}{dx} - y = x$

(iii) $2xy\frac{dy}{dx} = x^2 + y^2$

(iv) $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$

(v) $\frac{dy}{dx} + 2xy = 2x$

(b) Demand and supply functions are given by

$$D = 100 - P + \frac{dP}{dt}$$

$$S = -50 + 2P + 10\frac{dP}{dt}$$

Find the time path of P for dynamic equilibrium if the initial price is given to be ₹ 10 [i.e., when $t = 0$, $P(0) = 10$]. What will be the price at time $t = 10$? 4+2=6

9. Solve the following demand-supply model :

$$Q_{dt} = \alpha - \beta P_t$$

$$Q_{st} = -\gamma + \delta P_{t-1}$$

$$Q_{dt} = Q_{st}$$

Determine the time path and discuss its nature using diagrams. 9+6=15

UNIT—IV

10. (a) What is linear programming? 2

(b) Explain the concepts of—

(i) objective function;

(ii) constraints;

(iii) slack and surplus variables;

(iv) feasible solution and feasible region. 2+2+2+2=8

(c) (i) State the Hawkins-Simon conditions for the viability of the system in input-output analysis. 2

(ii) The input-output matrix is given by

$$A = \begin{bmatrix} 0\cdot3 & 0\cdot2 & 0\cdot2 \\ 0\cdot2 & 0\cdot1 & 0\cdot5 \\ 0\cdot2 & 0\cdot4 & 0\cdot2 \end{bmatrix}$$

Test the Hawkins-Simon conditions for viability of the system. 3

11. The input-output coefficients for a three-sector economy is given by

$$A = \begin{bmatrix} 0 & 0\cdot5 & 0 \\ 0\cdot2 & 0 & 0\cdot5 \\ 0\cdot4 & 0 & 0 \end{bmatrix}$$

The labour days required per unit of output are 0·4, 0·7 and 1·2 respectively. Consumer output targets (i.e., final demand) are 1000, 5000 and 4000 respectively.

Find—

(a) gross output for each sector;

(b) total labour days required;

(c) total value added or cost of primary input (labour);

(d) equilibrium price. 6+2+2+5=15

12. *(a)* Solve the following linear programming problem using graphical method :

$$\text{Maximize } Z = 45x_1 + 80x_2$$

subject to constraints

$$5x_1 + 20x_2 \le 400$$
$$10x_1 + 15x_2 \le 450$$
$$x_1,\ x_2 \ge 0$$

Also identify the feasible region. 11

(b) Write the dual of the following LPP : 2+2=4

(i) Maximize $Z = 3x_1 + 2x_2$

subject to constraints

$$5x_1 + 6x_2 \le 30$$
$$4x_1 + 7x_2 \le 25$$
$$8x_1 + 9x_2 \le 16$$
$$x_1,\ x_2 \ge 0$$

(ii) Minimize $Z = 2x_1 + 2x_2$

subject to constraints

$$2x_1 + 4x_2 \ge 1$$
$$x_1 + 2x_2 \ge 1$$
$$2x_1 + x_2 \ge 1$$
$$x_1,\ x_2 \ge 0$$

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